# The 2nd China-Russia Conference on Topology and Combinatorics

## Introduction:

The China-Russia Conference on Topology and Combinatorics aims to strengthen academic exchanges between China and Russia. The conference focuses on the frontier issues of topology and combinatorics. The 1st China-Russia Conference on Topology and Combinatorics was held by Northeastern University from March 15 to 20, 2021. The 2nd China-Russia Conference on Topology and Combinatorics will be co-hosted by Yanqi Lake Beijing Institute of Mathematical Sciences and Applications (BIMSA) and Hebei Normal University in collaboration with Moscow Institute of Physics and Technology (MIPT) from December 22 to 25, 2022.

You can participate online through the following ZOOM ID:
ZOOM ID: 388 528 9728 PW: BIMSA

## Plenary speakers:

Combinatorics：
Andrei M. Raigorodskii (Moscow Institute of Physics and Technology)
Janos Pach （Federal Institute of Technology in Lausanne）
Liping Yuan (Hebei Normal University)
Maksim Zhukovskii (Moscow Institute of Physics and Technology)
Sergei Lando (Higher School of Economics)

Topology:
Andrei Vesnin（Tomsk State University）
Jie Wu (BIMSA)
Louis Kauffman （ University of Illinois at Chicago ）
Mikhail Khovanov (Columbia University)
Roger Fenn（University of Sussex）
Sergei Gukov （California Institute of Technology）
Sofia Lambropoulou （National Technical University of Athens）
Vassily Manturov（Moscow Institute of Physics and Technology）

## Academic Committee:

Louis Kauffman (University of Illinois at Chicago)
Sergei Gukov (California Institute of Technology)
Sergei Matveev (Chelyabinsk State University)
Igor Nikonov (Lomonosov Moscow State University)
Vassily Manturov (Moscow Institute of Physics and Technology)
Andrei M. Raigorodskii (Moscow Institute of Physics and Technology)

Jie Wu (BIMSA)
Liping Yuan (Hebei Normal University)

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Sergei Ivanov (BIMSA)

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Huyue Yan (Eberhard-Karls-Universitaet Tuebingen)

# Schedule

### Schedule of talk(in preparation)

| | | |
|---|---|---|
| **Thursday 22 December** | | |
| Morning Session 9:00-11:30 (Beijing time) Chair: **Jiajun Wang** | | |
| 9:00-10:00 | **Mkhail Khovanov** | Automata and topological theories |
| 10:30-11:30 | **Louis Kauffman** | Mock Alexander Polynomials, Starred Knots, Knotoids and Virtual Knots |
| Afternoon Session 14:00-18:00 (Beijing time) Chair: **Andrei Vesnin** | | |
| 14:00-15:00 | **Jie Wu** | Homotopy Patterns in Combinatorial Groups and Knot Theory |
| 15:20-15:50 | ***Yulan Qing*** | Boundary of Groups |
| 16:10-16:40 | ***Yu Pan*** | Augmentations and Exact Lagrangian surfaces |
| 17:00-18:00 | **Roger Fenn** | Generalised Knots: the state of play |
| **Friday 23 December** | | |
| 13:00-18:00 (Beijing time) Chair: **Igor Nikonov** | | |
| 13:00-14:00 | **Sergei Gukov** | Graphs, Quivers, and Quantum Topology at generic q |
| 14:15-14:45 | **Ye Liu** | Holonomy Lie algebra of a geometric lattice |
| 15:00-16:00 | **Sofia Lambropoulou** | Passing from plat to standard closure of braids and vice-versa |
| 16:15-17:15 | **Janos Pach** | Perfection and Geometry |
| 17:30-18:00 | ***Xiaoming Du*** | $\mathbb{Z}_2$-Thurston norms for Seifert manifolds |
| **Saturday 24 December** | | |
| 14:00-18:00 (Beijing time) Chair: **Vassily Manturov** | | |
| 14:00-15:00 | **Andrei Raigorodskii** | Random subgraphs |
| 15:30-16:30 | **Sergei Lando** | Weight systems related to Lie algebras |
| 17:00-18:00 | **Liping Yuan** | On $\mathscr{F}$-convexity and related problems |
| **Sunday 25 December** | | |

| 14:00-18:00 (Beijing time) Chair: **Sergei Matveev** | | |
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| 14:00-15:00 | **Vassily Manturov** | Virtual knot theory methods in classical knot theory |
| 15:10-15:40 | **Igor Nikonov** | On skein invariants |
| 15:50-16:50 | **Maksim Zhukovskii** | Weak saturation in complete and random graphs |
| 17:00-18:00 | **Andrei Vesnin** | Combinatorics and volumes of hyperbolic polyhedra |

**Xiaoming Du**

**Title:** $\mathbb{Z}_2$-Thurston norms for Seifert manifolds

**Abstract:**

Given a manifold $M$ and an element $c \in H_2(M;\mathbb{Z}_2)$, the $\mathbb{Z}_2$-Thurston norm of $c$ is related to the minimal genus of the non-orientable surfaces representing the homology class $c$. The norms of all of the elements in the $\mathbb{Z}_2$-homology group can produce a lower bound of the complexity of $M$, i.e., the number of tetrahedra necessary to triangulate $M$. In this talk, for every orientable Seifert manifold $M$ with an orientable orbit space, we give a general method to compute the $\mathbb{Z}_2$-Thurston norms of all the elements in $H_2(M;\mathbb{Z}_2)$.

**Roger Fenn**

**Title:** Generalised Knots: the state of play

**Abstract:** Various different knot theories have been discovered (invented?) over the last few decades. We can make a list of properties/invariants associated with classical knot theory and try and find analogues for the new theories. These theories and their properties/invariants when found can be described combinatorially or geometrically. Some, such as singular knots use both descriptions. Some such as the fundamental group can be described combinatorially but so far not geometrically. Others such as crookedness have no analogue. In this talk I will try, in the time allotted, to describe some of this.

**Sergei Gukov**

**Title:** Graphs, Quivers, and Quantum Topology at generic q

**Abstract:** In this talk I will describe how developments in the past year lead to a curious relation between two combinatorial structures. One combinatorial structure is a graph whose vertices are decorated by integers --- or, equivalently, its adjacency matrix --- that has a meaning in topology. The second combinatorial data is similar, except it describes the connectivity structure of a quiver and has a meaning in representation theory of vertex algebras. Hence, we obtain a curious relation between topology and vertex algebras that can be expressed in a simple language of graph / quiver combinatorics.

**Louis H Kauffman**

**Title:** Mock Alexander Polynomials, Starred Knots, Knotoids and Virtual Knots

**Abstract:**

This talk is joint work with Neslihan Gugumcu.

A "starred knot" is a knot or link diagram with "stars" placed in some of its regions. Reidemeister moves are not allowed to pass arcs across the stars. The starred knot is taken up to this restricted form of Reidemeister equivalence. Adding the star is equivalent to removing a tube from either the thickened plane or from the thickened two-sphere (or from a thckened surface when we generalize to virtual knots). Thus we are using diagrammatic models for knots and links in handlebodies. This talk will discuss generalizations of the Alexander-Conway polynomial to starred knots, knotoids and knots in thickened surfaces. These generalizations use state summations that can be expressed in terms of permanents of matrices associated with the diagrams of the starreed entities. These state summations generalize the structures in the author's monograph Formal Knot Theory that originally apply to the Alexander-Conway polynomial. Thus we create analogs of the Alexander-Conway polynomial via state summation and study these Mock Alexander Polynomials in a number of contexts. In many contexts, Mock Alexander Polynomials can detect chirality. We will discuss our present state of knowledge and the conjectures that we are pursuing about them.

**Mikhail Khovanov**
**Title:** Automata and topological theories
**Abstract:** We explain the relation between regular languages and automata, on one side,
and one-dimensional TQFTs and topological theories with defects, on the other.

Sofia Lambropoulou
**Title:** Passing from plat to standard closure of braids and vice-versa
**Abstract:**
Given a knot or link in the form of the plat closure of a braid, we describe an algorithm to obtain a braid representing the same link with the standard closure, and vice-versa. We analyze the three cases of links: in \(mathbb{R}^3\), in handlebodies and in thickened surfaces. We show that the algorithm is quadratic in the number of crossings and loop generators of the braid when passing from plat to standard closure, while it is linear when passing from standard to plat closure. The plat closure representation turns out to be particularly suitable for computing knot invariants.
This is joint work with Paolo Cavicchioli (U Modena)

**Sergei Lando**
**Title:** Weight systems related to Lie algebras
**Abstract:**
V. A. Vassiliev's theory of nite type knot invariants allows one to associate to such an invariant a function on chord diagrams, which are simple combinatorial objects, consisting of an oriented circle and a tuple of chords with pairwise distinct ends in it. Such functions are called weight systems. According to a Kontsevich theorem, such a correspondence is essentially one-to-one: each weight system determines certain knot invariant. In particular, a weight system can be associated to any semi-simple Lie algebra. However, already in the simplest nontrivial case, the one for the Lie algebra sl(2), computation of the values of the corresponding weight system is a computationally complicated task. This weight system is of great importance, however, since it corresponds to a famous knot invariant known as the colored Jones polynomial. The last year was a period of signicant progress in understanding and computing Lie algebra weight systems, both for sl(2)- and gl(N)-weight system, for arbitrary N. New recurrence relations were deduced, which allow for a lot of explicit formulas. These methods are based on an idea, due to M. Kazarian, which suggests to extend the gl(N)-weight system to permutations. Questions concerning possible integrability properties of the Lie algebra weight systems will be formulated. The talk is based on work of M. Kazarian, the speaker, and the students P. Zakorko, Zhuoke Yang, and P. Zinova.

**Ye Liu**
**Title:** Holonomy Lie algebra of a geometric lattice
**Abstract:**

Holonomy Lie algebra is introduced by Kuo-Tsai Chen in his study of iterated integrals. Motivated by an interesting result of Kohno, which gives a combinatorial description of the holonomy Lie algebra of the complement to a complex hyperplane arrangement, we study the holonomy Lie algebra defined from a geometric lattice. We derive the relation of the holonomy Lie algebras of a solvable lattice pair. As applications, we obtain the holonomy Lie algebra structures of supersolvable (oriented) matroids and hypersolvable arrangements, as well as their lower central series formulae. Joint work with Weili Guo.

**Vassily Manturov**
**Title:** Virtual knot theory methods in classical knot theory (a joint work with I.M.Nikonov)
**Abstract:**
Over the last 20 years, virtual knot theory has experienced lots of new powerful invariants, including picture-valued invariants: invariants which are valued in linear combination of knot diagrams which sometimes allow one to judge about lots of properties of a knot by looking at one particular knot diagram. These constructions did not work in classical knot theory directly. In the present talk we map knots and braids in the thickened cylinder to virtual objects which allows one to pull back powerful knot invariants. The construction for braids allows one to upgrade the Burau representation to a stronger one by using virtual knot theory method.
The main construction takes a diagram on a cylinder (torus) and adds some ``invisible'' crossings which gives rise to a diagram which can be formally immersed but not embedded (drawn) on the cylinder (torus) and living comfortably in thickened surfaces of higher genera. This allows one to ``pull back'' invariants of virtual theory to the theory of knots in the thickened cylinder (torus).
The approach also allows one to study concordance of two-component links with one trivial component by using virtual knot theory methods.
V.O.Manturov, I.M.Nikonov, Maps to braids to virtual braids and braid representations, https://arxiv.org/abs/2210.06862
V.O.Manturov, I.M.Nikonov, Maps from knots in the cylinder to flat-virtual knots, https://arxiv.org/abs/2210.09689

**Igor Nikonov**
**Title:** On skein invariants
**Abstract:**
After J.H. Conway, it is known that some knot invariants can be defined by relations (called skein relations) on diagrams which differ at a local site. Among skein invariants one can mention Alexander and Jones polynomials, Arf invariant and writhe polynomial. In the talk we will remind these and other examples of skein invariants and introduce a new skein invariant for links in a fixed thickened surface.

**Janos Pach**

**Title:** Perfection and Geometry

**Abstract:** The chromatic number of every graph is at least as large as its clique number. For perfect graphs, these two numbers coincide. Most graphs are far from perfect, but many geometrically defined graphs are "nearly perfect" in the following sense: their chromatic numbers are bounded by a function f of their clique numbers. How far these graphs are from being perfect, can be measured by the growth rate of f. After giving a whirlwind tour of the subject, we outline the proof of some new results related to arrangements of curves in the plane, and we mention some open problems.

**Yu Pan**

**Title:** Augmentations and Exact Lagrangian surfaces

**Abstract:**

A major theme in symplectic and contact topology is the study of Legendrian knots and exact Lagrangian surfaces. In the talk, we will talk about some flexibility results of immersed Lagrangian surfaces using augmentation, a Floer type invariant of Legendrian knots. In particular, for an immersed filling of a topological knot, one can do surgery to resolve a double point with the price of increasing the surface genus by 1. In the Lagrangian analog, one can do Lagrangian surgery on immersed Lagrangian fillings to treat a double point by a genus. In this talk, we will explore the possibility of reversing the Lagrangian surgery, i.e., compressing a genus into a double point. It turns out that not all Lagrangian surgery is reversible.

**Yulan Qing**

**Title:** Boundary of Groups

**Abstract:**

Gromov boundary provides a useful compactification for all infinite-diameter Gromov hyperbolic spaces. It consists of all geodesic rays starting at a given base-point and it has been an essential tool in the study of the coarse geometry of hyperbolic groups. In this study we introduce a generalization of Gromov boundary for all finitely generated groups. We construct the sublinearly Morse boundaries and show that it is a QI-invariant topological space and it is metrizable. We show the geometric genericity of points in this boundary using Patterson Sullivan measure on the visual boundary of CAT(0) spaces. Lastly we discuss the connection between the sublinearly Morse boundary and random walk on groups. As applications we answer open problems regarding QI-invariant models of random walk on CAT(0) groups and on the mapping class group. This talk is based on several joint projects with Kasra Rafi and Giulio Tiozzo.

**Andrei Raigorodskii**

**Title:** Random subgraphs

**Abstract:**

I'll give a survey of classical and recent results on some extremal characteristics of random subgraphs of some graphs appearing in geometry and coding theory.

**Andrey Vesnin**

**Title:** Combinatorics and volumes of hyperbolic polyhedra

**Abstract:**

In 1922 Steinitz characterized graphs arising as one-dimensional skeletons of three-dimensional convex polyhedra. In 1970 Andreev obtained necessary and sufficient conditions for a polyhedon with given combinatorial type and acute dihedral angles to be realized in a hyperbolic three-dimensional space. This realization is unique up to isometry. A hyperbolic polyhedron is said to be right-angled if all dihedral angles are equal to pi/2. The class of bounded right-angled polyhedra contains fullerenes. The initial list of bounded right-angled polyhedra with their volumes was described by Inoue (2015). The class of ideal right-angled polyhedra is useful for construing hyperbolic structure on link complements. The initial list of ideal right-angled polyhedra with their volumes was presented by Vesnin and Egorov (2020). We will discus upper bounds of volumes of right-angled polyhedra which depend of number of vertices only. The talk is based on joint paper with of S. Alexandrov, N. Bogachev and A. Egorov. (Preprint version is available at https://arxiv.org/abs/2111.08789).

**Jie Wu**

**Title:** Homotopy Patterns in Combinatorial Groups and Knot Theory

**Abstract:**

The notion of homotopy pattern was recently introduced by Roman Mikhailov in the Proceedings of the ICM 2022. In this talk, we will give an introduction to the homotopy patterns in group theory proposed by Mikhailov as well as a recent solution of Laurent Bartholdi and Roman Mikhailov to a longstanding dimension problem in group theory using homotopy theory. Moreover, we will also give an introduction to some recent works on homotopy patterns in knot theory contributed by people in Chinese school of topology. Particularly, we will introduce a recent work of Yu Zhang on the topic.

**Liping Yuan**

**Title:** On $\mathscr{F}$-convexity and related problems

**Abstract:** Let $\mathscr{F}$ be a family of sets in IR$d$ . A set $M \subset \mathscr{F}$ is called $\mathscr{F}$-convex if for any pair of distinct points $x, y \in M$, there is a set $F \in \mathscr{F}$ such that $x, y \in F$ and $F \subset M$. In this talk we'll discuss $\mathscr{F}$-convexity and related problems for some interesting families $\mathscr{F}$.

**Maksim Zhukovskii**

**Title:** Weak saturation in complete and random graphs

**Abstract.** Let F,G be two graphs. Consider a monotone cellular automaton (known as edge bootstrap percolation) defined on the edges of G in the following way. Start from a spanning subgraph H of G, and on every step, add a single edge that produces at least one new copy of F. The minimum number of edges in H such that the bootstrap percolation reaches the entire set of edges of G is called the weak saturation number wsat(G,F). It was conjectured by Bollobas that for G,F being cliques of size n and s respectively, the optimal H to start from is obtained by deleting from G a subclique of size n-s+2. This conjecture was proven in several papers using algebraic methods, and the first proof was given by Lovasz. In the talk, we will concentrate on two particular cases: G is a clique and G is a random graph. Even when G is a clique, the exact value (and even the asymptotics) of wsat(G,F) is not known in general (for arbitrary F). However, we received new bounds that give at least sharp asymptotic results for some families of pattern graphs F. Quite recently, Korandi and Sudakov proved that with high probability wsat(G,K_s)=wsat(K_n,K_s) when G is a binomial random graph with constant edge probability. We conjecture that this is true for all pattern graphs

and give some arguments in the favour of this conjecture. As the random graph becomes sparser (i.e. the edge probability p, that in general depends on the number of vertices of the random graph, becomes smaller), its weak saturation number changes. We will discuss thresholds for the stability property - i.e. the minimum value of p such that wsat(G,K_s)=wsat(K_n,K_s) with probability bounded away from 0. This question is not solved even for s=3, and it turns out that it is highly related with the threshold probability for vanishing of the fundamental group of the 2-dimensional simplicial complex of the random graph.